STEADY-STATE ANALYSIS OF COUPLED-INDUCTOR ĆUK PWM CONVERTER
Part I: Continuous Conduction Mode

Elena Niculescu
Department of Electronics and Instrumentation
University of Craiova
Al. I. Cuza Street, No. 13, Craiova
ROMANIA
email: eniculescu@electronics.ucv.ro

Abstract: The paper presents a steady-state analysis of Ćuk PWM converter with the inductors coupled for continuous conduction mode (CCM). The results of this simplified analysis allow to determine the operating point of converters and further the small-signal low-frequency parameters of linear model, and to design the converter too. The expressions of all steady-state currents and voltages take the effects of coupling into account. The converter with separate inductors is treated as a particular case of the more general coupled-inductor case.

Key-Words: Steady-state analysis, Coupled-inductor Ćuk PWM converter, Continuous conduction mode

Nomenclature
i_{L1} = inductor current in L_1
i_1 = ripple component of inductor current in L_1
I_{L01} = initial and final conditions of inductor current in L_1
I_{L1D} = average value of ripple component of inductor current in L_1
I_{L1} = average inductor current in L_1
i_{L2} = inductor current in L_2
i_2 = ripple component of inductor current in L_2
I_{L02} = initial and final conditions of inductor current in L_2
I_{L2D} = average value of ripple component of inductor current in L_2
I_{L2} = average inductor current in L_2
I_I = averaged input current
I_O = averaged output current
i_{SW} = current in transistor switch SW
i_{Df} = current in flywheel diode D_f
V_I = dc input voltage
V_O = dc output voltage
v_{L1} = inductor voltage across L_1
v_{L2} = inductor voltage across L_2
V_{C1} = averaged voltage across the energy storage capacitor C_1
D_1 = duty cycle
D_2 = duty cycle of flywheel diode
f_s = switching frequency
L_1 = inductance of filtering inductor L_1
L_2 = inductance of filtering inductor L_2
L_M = mutual inductance between L_1 and L_2
k_c = coupling coefficient between L_1 and L_2
n = turns ratio
C_1 = capacity of energy storage capacitor C_1
C_2 = capacity of output filtering capacitor C_2
η = conversion efficiency of converter
M = dc voltage conversion ratio

I. INTRODUCTION

Like many areas of engineering, power electronics is mainly motivated by practical applications and it often turns out that a particular circuit topology long before it has been thoroughly analysed. For instance, the Ćuk PWM converter uncovered in 1977 (Ćuk 1977) fast becomes the fourth basic configuration close by well-known buck, boost and buck-boost PWM converters. However, good analytical models allowing a better understanding and systematic circuit design was only developed in the late 1980s and in-depth analytic characterisation and modelling is still being actively pursue today. Various modelling approaches applied to Ćuk PWM converter with both continuous and discontinuous conduction modes and more refined models of this converter length-ways have been reported (Chetty 1983; Vorperian 1990; Lee et al. 1992).

As it is well known, the PWM converters are non-linear dynamic systems with structural changes over an operation cycle: two for the continuous conduction mode (CCM) and three for the discontinuous conduction mode (DCM). Coupled inductor techniques can be applied to Ćuk PWM converter operating either with CCM or DCM to achieve ripple-free input or output current (Ćuk 1977; Niculescu 2002).

The purpose of this paper is to provide a complete steady-state characterization for coupled-inductor Ćuk PWM converter with both CCM and DCM. The converter with separate inductors is treated as a particular case of the more general coupled-inductor case. In Section II, the steady-state analysis of Ćuk PWM converter with CCM is made. The expressions of averaged input and output currents, output voltage and voltage across energy storage capacitor, taking the effects of coupling into account, are derived here. A mathematical model for the steady-state behavior of converter has been obtained as result of this analysis based on the equations written on the circuit for each time interval corresponding to states of switches (transistor and diode) and the waveforms of electrical quantities corresponding to continuous operating mode of converter.

II. STEADY-STATE ANALYSIS OF ĆUK CONVERTER WITH CCM

The diagram of converter with coupled inductor is given in Fig. 1. Assume that the converter operates in continuous conduction mode and that the two inductors are magnetically coupled together to reduce the current ripples. The equivalent circuit of two coupled inductors is given in Fig. 2, and the waveforms of inductor currents and voltages are shown in Fig. 3 where $D_2 = 1 - D_1$.

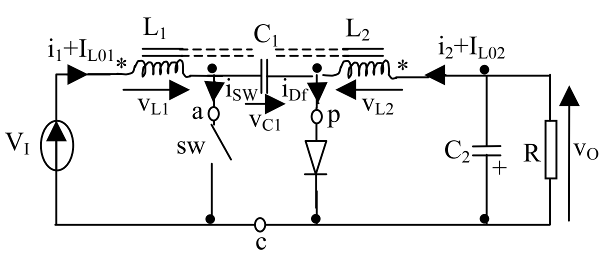


Fig.1. The diagram of Ćuk PWM converter with coupled inductors

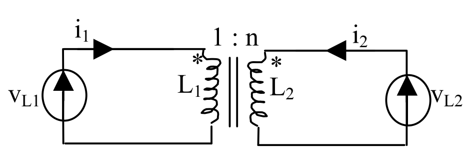
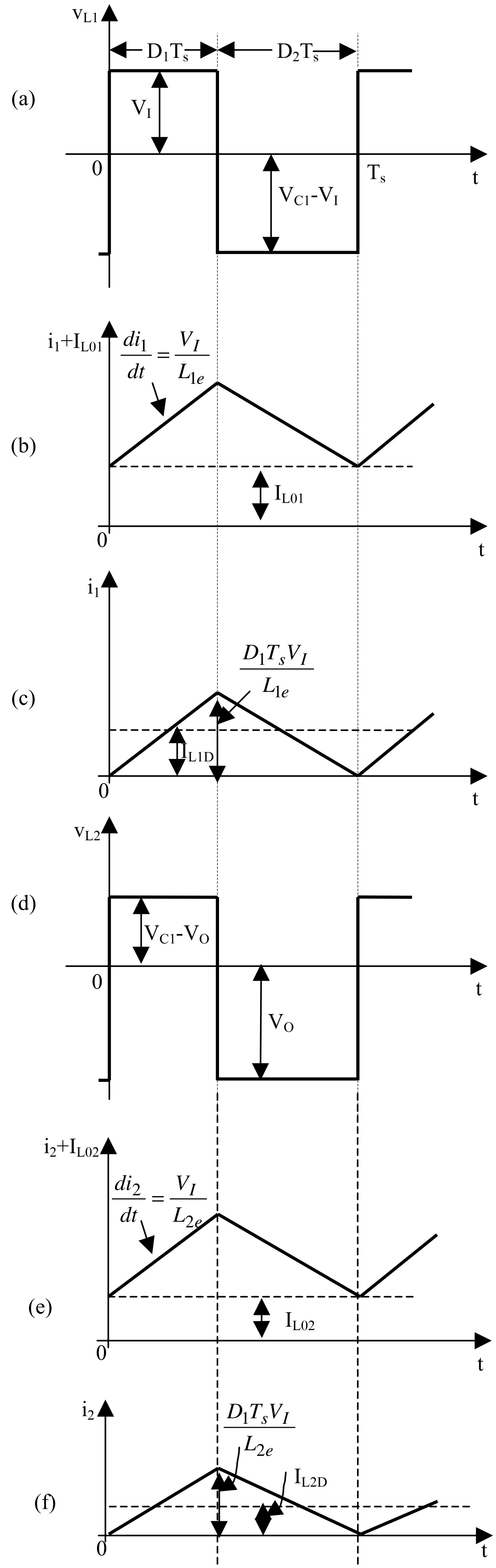


Fig.2. Equivalent circuit of two coupled inductors

The target of this analysis is that to find the expressions of dc voltage conversion ratio, averaged inductor currents, averaged input and output currents, averaged voltage across the energy storage capacitor C_1.



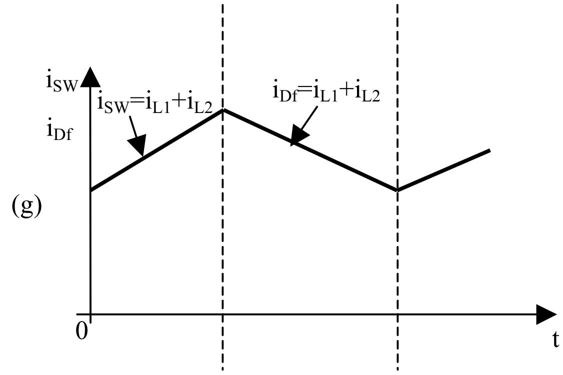


Fig.3. The waveforms of inductor currents and voltages for continuous conduction mode

As it can be seen from the inductor current waveforms given in Fig. 3 (a) and (d), the ripple component i_1 respectively i_2 is superposed over the dc component I_{L01} respectively I_{L02} that represent the initial and final conditions of inductor current in L_1 respectively L_2:

$$i_{L1} = i_1 + I_{L01} \tag{1}$$
$$i_{L2} = i_2 + I_{L02} \tag{2}$$

In order to determine the averaged values of the current components, we need to find di_1/dt and di_2/dt firstly. The input and output voltages as well as the voltage across the energy storage capacitor C_1 are supposed without ripple. Based on the equivalent circuit of two coupled inductors from the Fig.2, we have

$$v_{L1} = L_1 \frac{di_1}{dt} + L_M \frac{di_2}{dt} \tag{3}$$
$$v_{L2} = L_M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \tag{4}$$

Solving for the unknown di_1/dt and di_2/dt, we find

$$\frac{di_1}{dt} = v_{L1} \frac{L_2}{L_1 L_2 - L_M^2} - v_{L2} \frac{L_M}{L_1 L_2 - L_M^2} \tag{5}$$
$$\frac{di_2}{dt} = v_{L2} \frac{L_1}{L_1 L_2 - L_M^2} - v_{L1} \frac{L_M}{L_1 L_2 - L_M^2} \tag{6}$$

After some algebra, the previous equations can be written as

$$\frac{di_1}{dt} = \frac{v_{L1} - \frac{k_c}{n} v_{L2}}{\left(1 - k_c^2\right) L_1} \tag{7}$$
$$\frac{di_2}{dt} = \frac{v_{L2} - k_c n v_{L1}}{\left(1 - k_c^2\right) L_2} \tag{8}$$

For $0 \le t \le D_1 T_s$, by substitution of inductor voltages $v_{L1} = V_I$ and $v_{L2} = V_{C1} - V_O$, the equation (7) and (8) yield

$$\frac{di_1}{dt} = \frac{V_I - \frac{k_c}{n}\left(V_{C1} - V_O\right)}{\left(1 - k_c^2\right) L_1} \tag{9}$$
$$\frac{di_2}{dt} = \frac{\left(V_{C1} - V_O\right) - k_c n V_I}{\left(1 - k_c^2\right) L_2} \tag{10}$$

We proceed similarly for $D_1 T_s \le t \le T_s$ when $v_{L1} = -\left(V_{C1} - V_I\right)$ and $v_{L2} = -V_O$, and we obtain

$$\frac{di_1}{dt} = -\frac{\left(V_{C1} - V_O\right) - \frac{k_c}{n} V_O}{\left(1 - k_c^2\right) L_1} \tag{11}$$
$$\frac{di_2}{dt} = -\frac{V_O - k_c n\left(V_{C1} - V_O\right)}{\left(1 - k_c^2\right) L_2}. \tag{12}$$

Since the total rise of i_1 and respectively i_2 during $0 \le t \le D_1 T_s$ must to be equal to the total fall of the same currents during $D_1 T_s \le t \le T_s$, we have

$$D_1 T_s \times \left[\frac{di_1}{dt}\right]_{0 \le t \le D_1 T_s} = \left(1 - D_1\right) T_s \times \left[-\frac{di_1}{dt}\right]_{D_1 T_s \le t \le T_s} \tag{13}$$

$$D_1 T_s \times \left[\frac{di_2}{dt}\right]_{0 \le t \le D_1 T_s} = \left(1 - D_1\right) T_s \times \left[-\frac{di_2}{dt}\right]_{D_1 T_s \le t \le T_s}. \tag{14}$$

Substituting (9), (10) and (11), (12) into (15) and (16) yields the steady-state relationships between the DC input voltage and the averaged output voltage, and the averaged voltage across the energy storage capacitor C_1:

$$V_{C1} = \frac{V_I}{1 - D_1}; \; V_O = \frac{D_1 V_I}{1 - D_1}.$$

From these results, the dc voltage conversion ratio of coupled-inductor Ćuk PWM converter with CCM is obtained as

$$M = \frac{V_O}{V_I} = \frac{D_1}{1 - D_1} \tag{15}$$

and

$$V_{C1} = V_I + V_O. \tag{16}$$

The above results have been obtained even for the separate inductor case. So, the magnetically coupling of inductors do not modifies the dc voltage conversion ratio that only depends on the duty cycle of PWM converter and also the relationship between the voltages V_{C1}, V_O and V_I. On the other hand, the relationship (16) shows that the voltages across the two inductors are equal during each distinct time interval that allows to couple the inductors in order to reduce the input or output current ripple (Ćuk 1977; Niculescu 2002).

Using these new results, the equations (9), (10) and (11), (12) become

$$\frac{di_1}{dt}=\frac{V_I}{L_{1e}} \tag{17}$$

$$\frac{di_2}{dt}=\frac{V_I}{L_{2e}} \tag{18}$$

and

$$\frac{di_1}{dt}=-\frac{V_O}{L_{1e}} \tag{19}$$

$$\frac{di_2}{dt}=-\frac{V_O}{L_{2e}} \tag{20}$$

where we denoted two effective inductances

$$L_{1e}=\frac{\left(1-k_c^2\right)L_1}{1-\dfrac{k_c}{n}} \tag{21}$$

$$L_{2e}=\frac{\left(1-k_c^2\right)L_2}{1-k_c n}. \tag{22}$$

The above results are shown on the waveforms of the inductor currents given in the Fig. 3 from which the averaged ripple component of inductor currents can be found as:

$$I_{L1D}=\frac{D_1T_sV_I}{2L_{1e}}=\frac{D_1V_I}{k_{1m}R} \tag{23}$$

$$I_{L2D}=\frac{D_1T_sV_I}{2L_{2e}}=\frac{D_1V_I}{k_{2m}R} \tag{24}$$

where we denoted two parameters of conduction through the inductors as follows:

$$K_{1m}=\frac{2L_{1e}f_s}{R} \tag{25}$$

$$K_{2m}=\frac{2L_{2e}f_s}{R} \tag{26}$$

Next, we determine the components I_{L01} and I_{L02} of inductor currents. For this purpose, we use the equations given below:

$$I_O=I_{L2}=I_{L2D}+I_{L02}=\frac{V_O}{R}=\frac{MV_I}{R} \tag{27}$$

$$V_O\left(I_{L2D}+I_{L02}\right)=\eta V_I\left(I_{L1D}+I_{L01}\right). \tag{28}$$

For a converter without losses, we obtain the following results:

$$I_{L01}=\frac{D_1V_I}{R}\left[\frac{D_1}{(1-D_1)^2}-\frac{1}{K_{1m}}\right] \tag{29}$$

$$I_{L02}=\frac{D_1V_I}{R}\left[\frac{1}{1-D_1}-\frac{1}{K_{2m}}\right]. \tag{30}$$

We can now to find the averaged currents of transistor switch and diode as follows:

$$I_{SW}=D_1\left(I_{L1}+I_{L2}\right)=\frac{D_1^2V_I}{R(1-D_1)^2}=\frac{M^2V_I}{R} \tag{31}$$

$$I_{Df}=(1-D_1)\left(I_{L1}+I_{L2}\right)=\frac{D_1V_I}{R(1-D_1)}=\frac{MV_I}{R}. \tag{32}$$

The results of this simplified analysis characterize completely the steady-state behavior of the coupled-inductor Ćuk PWM converter in continuous conduction mode. For the separate inductor case, it is enough to set k_c at zero into the above results. As consequence, $L_{1e}\to L_1$, $L_{2e}\to L_2$, $K_{1m}\to K_1$ and $K_{2m}\to K_2$.

III. CONCLUSION

A steady-state analysis that characterizes completely the behavior of Ćuk PWM converter with coupled or separate inductors and operating in CCM was made in this paper. Taking the effects of coupling into account, the expressions of all steady-state currents such as the averaged input and output currents, and the averaged currents of transistor and diode are found. For the dc voltage conversion ratio and the averaged voltage across the energy storage capacitor, we found the same formula as in the separate inductor case, owing of the neglecting of equivalent series resistance of capacitors. The steady-state mathematical model developed here supplies the formula of equivalent inductance in the input and output circuits of converter that are entailed by a dynamic analysis based on the small-signal equivalent circuit of converter.

REFERENCES

Ćuk, S. and Middlebrook, R.D., A New Optimum Topology Switching DC-to-DC Converter, *PESC'77 Record,* 1977, pp.160-179

Chetty, P.R.K., Modelling and Analysis of Ćuk Converter Using Current Injected Equivalent Circuit Approach, *IEEE Trans. on Industrial Electronics*, Vol. IE-30, No.1, 1983, pp. 56-59

Vorperian, V., Simplified analysis of PWM converters using model of PWM switch, Part I: Continuous conduction mode, *IEEE Trans. on Aerospace Electronic Systems*, Vol. 26, 1990, pp. 497-505

Lee, Y.S., Cheng, D.K. and Wong, S.C., A New Approach to the Modeling of Converters for SPICE Simulation, *IEEE Trans. on Power Electronics*, Vol. 7, No. 4, 1992, pp. 741-753

Niculescu, E., Comparative Analysis of DC/DC Fourth-Order PWM Converters, Annals of University of Craiova, Electrical Engineering Series, Year 26, No. 26, 2002, pp. 196 - 207